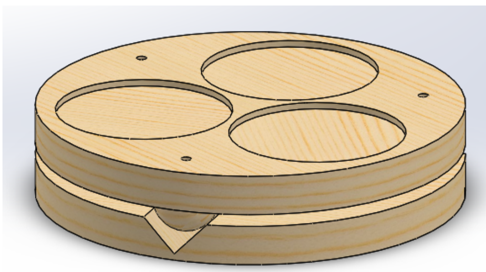
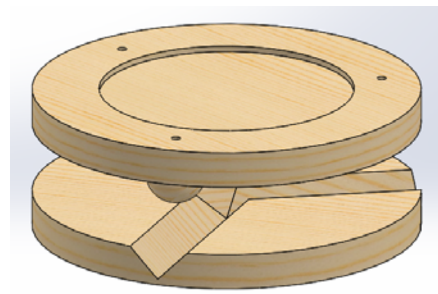
Kinematically Coupled Tea Set

1. Background

The goal for this project is to make a kinematically coupled wooden tea set placemat with interchangeable faceplates for mugs or a kettle. The kinematic coupling helps stabilize the tea set and allow for quick and convenient placement.



Mug placemat setting, showing coupled fit



Kettle placemat, showing grooved structure

Final product

1A. Kinematic Couplings

Kinematic couplings are a deterministic mechanism that are frequently used in precision engineering designs. By having the degrees of freedom equal to the number of points of constraint, a kinematic coupling forms an exact constraint between two objects, thus creating repeatable, predictable, and precise placement. Kinematic couplings tend to be cheaper to manufacture and design, albeit at the cost of higher contact stresses.

1B. Application

I have a friend who really enjoys drinking tea and hosting tea parties. However, after he uses his electric tea kettle to heat up the water, he does not have anywhere to place the kettle to have it cool off. Although there is insulation on the bottom of the kettle and the heating element is not exposed, the kettle still remains very hot to the touch and he would prefer to not set it directly on the table. He also does not have any coasters so water rings are left everywhere during a tea party.

This kinematically coupled tea set not only provides a central place to put the kettle and mugs, but also provides a stylish centerpiece for a party. The kinematic couple helps ensure that the set remains stable and also enables the top plate to be removed for quick clean up.

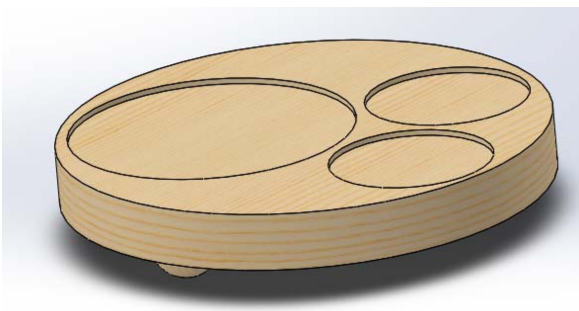
2. Design

Each placemat is made up of two wooden disks with a traditional three-groove kinematic coupling. The top disk either has insets cut out for the kettle or 3 mugs with the spheres affixed underneath, while the bottom disk had three v-grooves cut out.

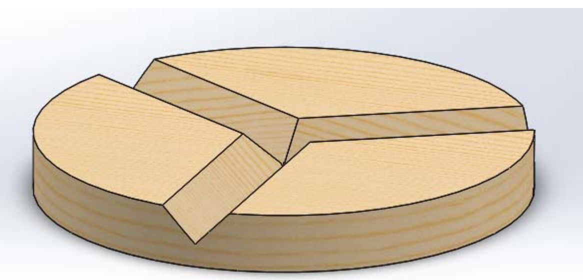
Many design changes occurred between proposal and final production, mainly due to the availability of materials and suggestions from the course instructors to speed up fabrication.

2A. Initial Design

Originally, the two disks were to be made out of pine wood, with diameter of 10.5” and a thickness of 1.25”. The top plate would have three 1.5” diameter wooden hemispheres glued to its base which fit into the v-grooves in the bottom plate. The top plate would also have .125” deep insets cut out to fit an induction tea kettle (6” in diameter) and 2 mugs (3.5” in diameter). Technical drawings can be seen in Appendix A, Section I.



Original top plate design



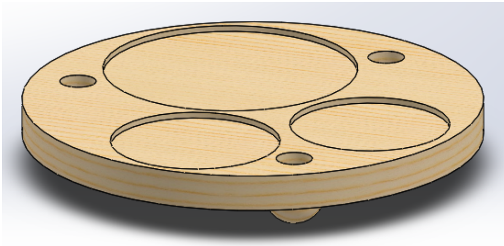
Original bottom plate design

2B. Post-Feedback Design

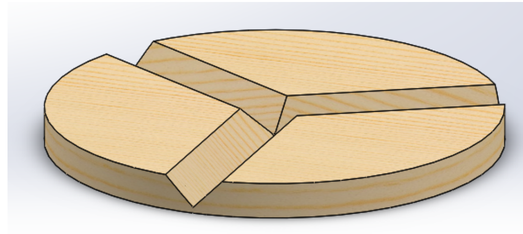
After submitting the above design, I got the OK to fabricate with the suggestions to consider using the V-groove jig as well as consider using a bandsaw rather than a router to cut out the main circle shape. These suggestions were made to enable faster fabrication.

This led me to more closely inspect the available materials that were actually at the hobby shop. Although the website said that there would be pine stock of any size available, in reality, only the 10” x 10” x 0.75” plywood squares were available. Thus, I had to shrink the size of the disks from 10.5” to 9.5” so that they would fit. The inset holes were also shrunk to accommodate the smaller disk size, sizing instead for a smaller induction kettle and a smaller mug.

The reduced height of the plywood squares also forced me to choose a smaller sphere size for the kinematic coupling as the 1.5” groove would mean that a V-groove would be too large for the plywood. I decided to use the 1.25” spheres instead which would be positioned by drilling ¾” hole equally around the disk to match the flat. A ¾” dowel would then be fit through the hole and glued in to position the spheres. Technical drawings can be seen in Appendix A, Section II.



Post-feedback top plate design



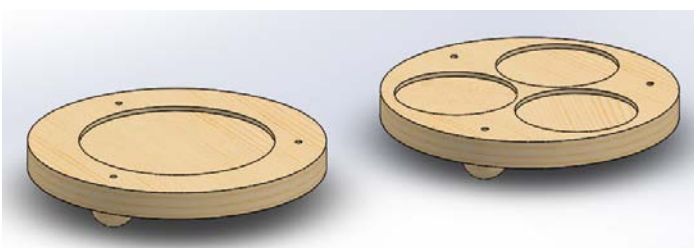
Post-feedback bottom plate design

2C. Final Design

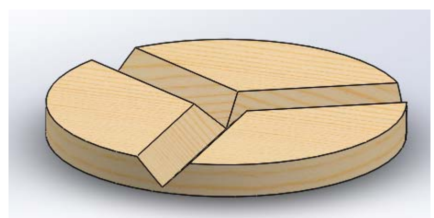
When I went to manufacture the above design, the original diameter of 9.5” was too large for the v-groove jig to cut. The disk diameter extended past the edges of the v-groove jig so we could no longer follow the jig’s edge to get our 120 degree grooves. I decided to shrink the size of the disk to 8” rather than attempt to use a CNC machine to make the v-grooves so that I could keep a short production time. However, this smaller diameter of 8” meant that it was impossible to accommodate the 5.5” diameter kettle and the 3.3” diameter mugs at the same time.

I thus decided to make interchangeable top plates in order to still preserve the initial goal of having a place to set down a tea kettle and mugs in an easy-to-setup manner.

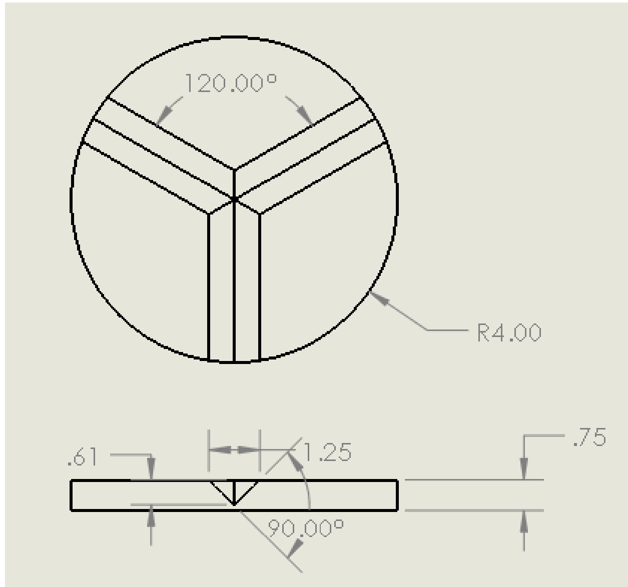
In addition, I found at the Hobby Shop that although the 2.75 website claimed that the dowels had diameter of 0.75”, they were actually 3/16”, so I redid those holes in the CAD while I was finalizing the design.



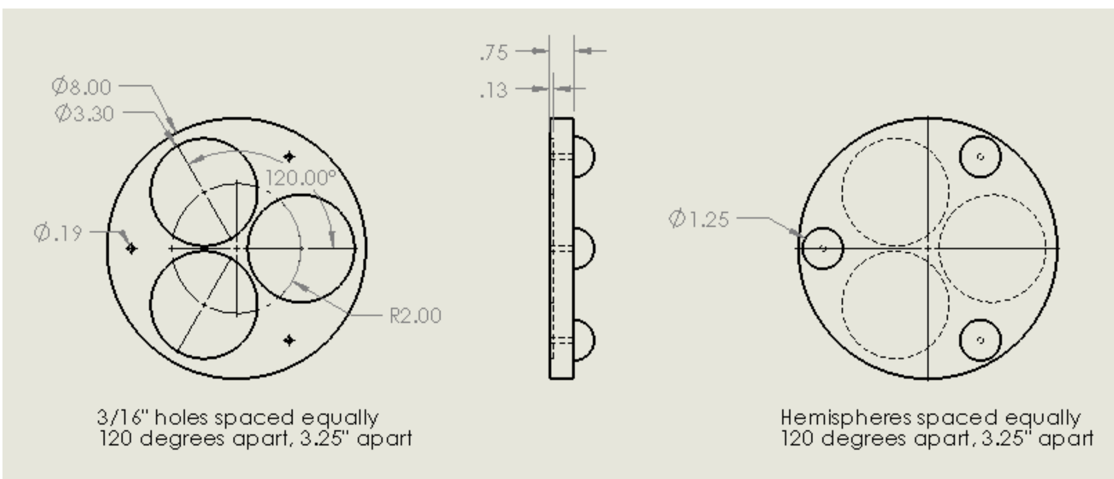
Kettle top plate and mug top plate of final design



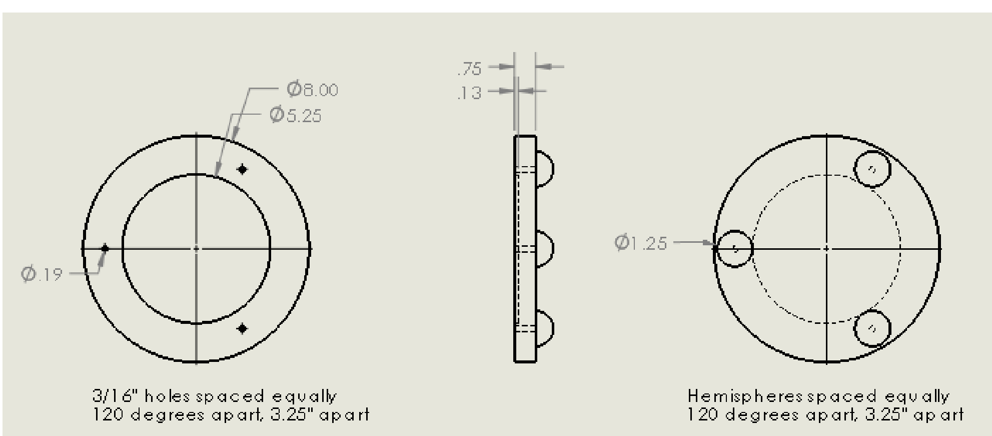
Final bottom plate design



Final bottom plate



Final top plate – mug setting



Final top plate – kettle setting

3. Manufacturing

3A. Final Fabrication Plan

Order	Part	Material	Process	Machine	Estimated Fab Time
1	Bottom	Wood	Drill guide hole for band saw jig	Drill press	5 min
2	Bottom	Wood	Cut into disc	Band saw + jig	10 min
3	Bottom	Wood	Cut V-grooves	Router table + jig	10 min
4	Top – kettle plate	Wood	Cut inset circles Cut circle out of square piece	CNC Router	1 hour programming + 20 minutes
5	Top – mug plate	Wood	Cut inset circles Cut circle out of square piece	CNC Router	1 hour programming + 20 minutes

3B. Fabrication Notes

Most of the manufacturing went according to plan, although there were a few deviations from the final design.

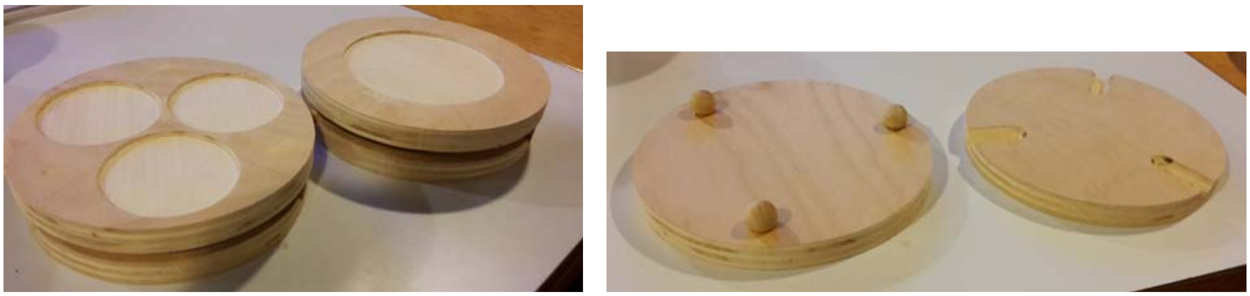
A small hole 3/16" in diameter, 0.25" deep was drilled in the center of each of the base plates in order to use the band-saw circle cutting jig. The hole is in a non-critical location as a center hole would not affect the critical kinematic coupling interface between the v-groove and the spheres.

In addition, I ended up using 0.75" spheres instead of my planed 1.25" spheres because the v-groove route rig in the Hobby Shop was set up for these smaller balls.

Using the CNC router was very smooth except for my last cut on the top kettle plate. For some reason, the Hobby Shop's version of MasterCAM would not produce the correct G-Code for contour cuts, so I had to use my own copy. I also had some trouble with the v-router table jig in cutting straight lines. At first, this was because I forgot to put double sided tape between my piece and the jig. Later, I also found issues with the setup as someone had moved the guides and not recentered it with the 120 degree jig.

Once the main mechanical part of the kinematic coupling was done, I added a few aesthetic touches. I used a laser cutter to raster some text on to all of the plates and also applied coats of stain and sealant to help prevent water damage to the parts. The engraving is not important to the kinematic coupling as all of the text was in noncritical locations. The stain and sealant may add some additional thickness to the v-groove and spheres. However, since a relatively even layer of thin coating was applied, the overall effect on repeatability and fit is probably minimal.

I had some difficulty in aligning the text of the top plates to the precise location, especially the "time for tea" pattern around the rim. I found out that this was because 1) the vector drawing program for the laser cutter did not respect the 1:1 measurement specified in my vector drawng program and 2) the laser cutter software would automatically remove whitespace, making positioning the cut piece with the laser's zero point very difficult. I fixed both problems by placing small dots at the corners of the 8x8 bounding rectangle which helped force the program to scale the image correctly.



Machined parts before stain

Machined parts after stain and laser engraving

4. Characterization

4A. Preliminary Analysis

To see the expected deflection and stresses that the kinematic coupling would take in normal operation, I used Alex Slocum's spreadsheet for three-groove couplings. Since I was using a traditional three-groove coupling with equally-spaced v-grooves, I did not need to change most of the values.

The most notable thing that changed in analysis from proposal to this writeup was the removal of the inclination angle. This last point was noted in proposal feedback and was the reason why I expected y-displacement when I was only applying a force in the –z direction.

Variable	Description	Value	Justification
Dbeq	Diameter of hemisphere	0.75 inches / 19.1 mm	
Rbminor / Rbmajor	Minor and major radius of contact of ball	0.375 inches / 9.53 mm	Ball is a perfect hemisphere so minor and major radii are the same
Dcoupling	Diameter of coupling circle	6.5 inches / 165 mm	
Fpreload	Preload applied over each ball	-6.67 N	Estimated weight of top plate (1.5 lbs) divided over 3 balls
(Xerr, Yerr, Zerr)	Location of where we are measuring error from centroid	(0, -101.6 mm, 0)	Will be using the laser pointer technique to measure deformation. We attached the laser pointer to the outside edge of the circle, so deflection is (0, -4 in, 0).
Wood – Yield Stress	Part of material properties	41.4 MPa	Flexural Yield Strength from http://www.matweb.com/search/DataSheet.aspx?MatGUID=7479536ff440400eae71cc721bf068c0
Wood – Elastic Modulus	Part of material properties	9 GPa	http://www.engineeringtoolbox.com/young-modulus-d_417.html
Wood – Poisson ratio	Part of material properties	0.34	Averaging out Poisson Ratios from: http://www.matweb.com/search/DataSheet.aspx?MatGUID=7479536ff440400eae71cc721bf068c0
Applied Z load at zero inclination – teapot	Force of the fully loaded tea plate	-25.4 N	Estimated weight of kettle weight (2 lbs) + water weight (1.69 kg for 57 fl oz) gives 5.7 lbs.
Applied Z load at zero inclination – mugs	Force of the fully loaded mug plate	-24 N	Estimated mug weight (3 x 1 lb) + water weight (3 x 350g for 350 mL) gives 5.3 lbs
Inclination angle (degrees)	Angle of the force applied	0 degrees	

The results of the spreadsheet analysis predicts that the total displacement for the teapot plate is (0, 0, 0.0019 mm) while the mug plate is (0, 0, 0.0018 mm). This is an acceptable amount of deflection since the tea set is not a high precision application.

The spreadsheet results in full is included below:
Spreadsheet result for teapot plate:

20	Matlabgroove =	4	where each ball and groove is defined individually			user defined	0.34			
21	Min. yield strength (Pa, psi)		4.14E+07	6,000						
22	Largest contact ellipse major diameter (mm)		0.495							
23	Smallest contact ellipse major diameter (mm)		0.494							
24	Largest contact stress ratio		1.21							
25	RMS applied force (N)		25.4							
26	RMS stiffness (N/micron)		13							
27	Z displacement caused by preload (mm)		0.003							
28	Applied Z load at zero inclinati	-24								
29	inclination angle (degrees)	0								
30	**Applied force's Z,Y,Z values and coordinates**				**Coupling centroid location**					
31	FLx (N) =	0.00	XL (mm) =	0	xc (mm)	0.000		XL (m) =	0.000	
32	FLy (N) =	0	YL (mm) =	0	yc (mm)	0.000		YL (m) =	0.000	
33	FLz (N) =	-25	ZL (mm) =	0	zc (mm)	0.000		ZL (m) =	0.000	
34	**Results: Hertz stresses and deformations**									
35	Error displacements at the point of interest due to applied load (preload displacement subtracted off) (micron)									
36	**DeltaX (mm)**	0.0000		**DeltaY (mm)**	0.0000		**DeltaZ (mm)**	0.0019		
37	**resulting rotation (degrees) w**	0.0000			0.0000					
38	**Vector displacement (mm)**	0.0019								
39	*roove normal forces (preload + applied load) (N*		*Contact stress (preload + applied load) (Pa)*		*Max shear stress/(ult.*	*Deflection (+into ball) (m)*			*Contact ellipse size (m)*	
40						*from:*	*Applied Load*	*Preload only*	*Rmajor*	*Rminor*
41	*Ball-Groove 1*		*Ball-Groove 1*			*Ball-Groove 1*			*Ball-Groove 1*	
42	Fbnone	*10.7*	sigone	*8.35E+07*	*1.210*	delone	*2.69E-06*	*3.70E-06*	*2.48E-04*	*2.47E-04*
43	Fbntwo	*10.7*	sigtwo	*8.35E+07*	*1.210*	deltwo	*2.69E-06*	*3.70E-06*	*2.48E-04*	*2.47E-04*
44	*Ball-Groove 2*		*Ball-Groove 2*			*Ball-Groove 2*			*Ball-Groove 2*	
45	Fbnthree	*10.7*	sigthree	*8.35E+07*	*1.210*	delthree	*2.69E-06*	*3.70E-06*	*2.48E-04*	*2.47E-04*
46	Fbnfour	*10.7*	sigfour	*8.35E+07*	*1.210*	delfour	*2.69E-06*	*3.70E-06*	*2.48E-04*	*2.47E-04*
47	Ball-Groove 3		Ball-Groove 3						Ball-Groove 3	
48	Fbnfive	*10.7*	sigfive	*8.35E+07*	*1.210*	delfive	*2.69E-06*	*3.70E-06*	*2.48E-04*	*2.47E-04*
49	Fbnsix	*10.7*	sigsix	*8.35E+07*	*1.210*	delsix	*2.69E-06*	*3.70E-06*	*2.48E-04*	*2.47E-04*

Spreadsheet result for mugs plate:

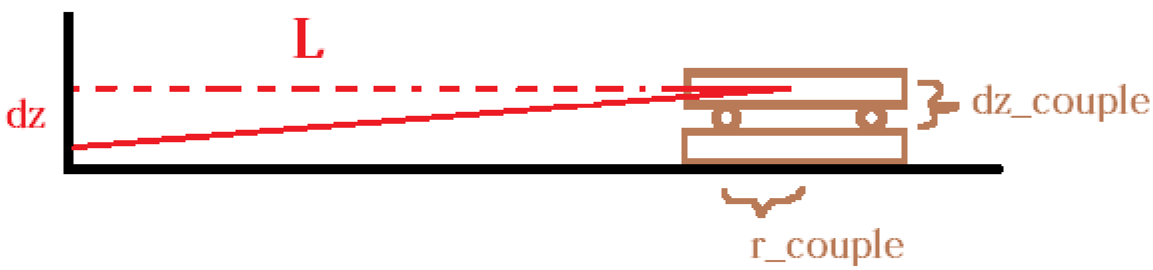
20	Matlabgroove =	4	where each ball and groove is defined individually			user defined	0.34			
21	Min. yield strength (Pa, psi)		4.14E+07	6,000						
22	Largest contact ellipse major diameter (mm)		0.490							
23	Smallest contact ellipse major diameter (mm)		0.489							
24	Largest contact stress ratio		1.20							
25	RMS applied force (N)		24							
26	RMS stiffness (N/micron)		13							
27	Z displacement caused by preload (mm)		0.003							
28	Applied Z load at zero inclinati	-24								
29	inclination angle (degrees)	0								
30	**Applied force's Z,Y,Z values and coordinates**				**Coupling centroid location**					
31	FLx (N) =	0.00	XL (mm) =	0	xc (mm)	0.000		XL (m) =	0.000	
32	FLy (N) =	0	YL (mm) =	0	yc (mm)	0.000		YL (m) =	0.000	
33	FLz (N) =	-24	ZL (mm) =	0	zc (mm)	0.000		ZL (m) =	0.000	
34	**Results: Hertz stresses and deformations**									
35	Error displacements at the point of interest due to applied load (preload displacement subtracted off) (micron)									
36	**DeltaX (mm)**	0.0000		**DeltaY (mm)**	0.0000		**DeltaZ (mm)**	0.0018		
37	**resulting rotation (degrees) w**	0.0000			0.0000					
38	**Vector displacement (mm)**	0.0018								
39	*roove normal forces (preload + applied load) (N*		*Contact stress (preload + applied load) (Pa)*		*Max shear stress/(ult.*	*Deflection (+into ball) (m)*			*Contact ellipse size (m)*	
40						*from:*	*Applied Load*	*Preload only*	*Rmajor*	*Rminor*
41	*Ball-Groove 1*		*Ball-Groove 1*			*Ball-Groove 1*			*Ball-Groove 1*	
42	Fbnone	*10.4*	sigone	*8.27E+07*	*1.198*	delone	*2.56E-06*	*3.70E-06*	*2.45E-04*	*2.45E-04*
43	Fbntwo	*10.4*	sigtwo	*8.27E+07*	*1.198*	deltwo	*2.56E-06*	*3.70E-06*	*2.45E-04*	*2.45E-04*
44	*Ball-Groove 2*		*Ball-Groove 2*			*Ball-Groove 2*			*Ball-Groove 2*	
45	Fbnthree	*10.4*	sigthree	*8.27E+07*	*1.198*	delthree	*2.56E-06*	*3.70E-06*	*2.45E-04*	*2.45E-04*
46	Fbnfour	*10.4*	sigfour	*8.27E+07*	*1.198*	delfour	*2.56E-06*	*3.70E-06*	*2.45E-04*	*2.45E-04*
47	Ball-Groove 3		Ball-Groove 3						Ball-Groove 3	
48	Fbnfive	*10.4*	sigfive	*8.27E+07*	*1.198*	delfive	*2.56E-06*	*3.70E-06*	*2.45E-04*	*2.45E-04*
49	Fbnsix	*10.4*	sigsix	*8.27E+07*	*1.198*	delsix	*2.56E-06*	*3.70E-06*	*2.45E-04*	*2.45E-04*

4B. Real-Life Analysis
In order to characterize the design, we decided to use a laser pointer to the tea set and exploit Abbe errors.

We attached a laser pointer to the outside rim of the tea set, so about 4 inches away from the center. We then set the tea set 12 feet away from a wall and put a piece of paper where the laser hit the wall. We then marked where the laser was without any load on the kinematic coupling. We then loaded the kinematic coupling with either a teapot full of water or 3 mugs full of water and then marked where the laser was again upon loading. We performed 3 trials for each top plate-bottom plate configuration.

After all trials were completed, we then measured the y and z deflection from the center. From this, we can get the angular deflection at the coupling point itself.

Let L be the distance from the wall and (dy, dz) be the deflection measured at the wall. We want to know what the deflection will be at the coupling point itself. Let the radius of the coupling circle be r_{couple} and the total deflection be (dy_{couple} , dz_{couple}). The following picture shows the relevant parameters for the z direction. An analogous picture exists for the y direction.



By similar triangles, it's clear to see that

$$dz_{couple} = \frac{dz \cdot r_{couple}}{L}$$

From this, we get the following results:

Teapot, Base 1			
y (mm)	z (mm)	dy_couple (mm)	dz_couple (mm)
-3.05	5.53	-0.0689	0.1249
2.06	6.17	0.0465	0.1393
4.14	6.44	0.0935	0.1454

Teapot, Base 2			
y (mm)	z (mm)	dy_couple (mm)	dz_couple (mm)
-1.63	2.07	-0.0368	0.0467
-2.7	2.07	-0.0610	0.0467
7.26	2.07	0.1639	0.0467

Teapot, Overall				
	Average dy_couple (mm)	Standard Deviation (mm)	Average dz_couple (mm)	Standard Deviation (mm)
Base 1	0.0237	0.0682	0.1365	0.0086
Base 2	0.0221	0.1008	0.0467	0.0000
Both	0.0229	0.0861	0.0916	0.0453

The upward tilt in the teapot measurements indicates that there is probably some unevenness in the sphere placement which is causing strange angular deflections.

Mugs, Base 1			
y (mm)	z (mm)	dy_couple (mm)	dz_couple (mm)
7.31	-3.77	0.1651	-0.0851
6.32	-5.48	0.1427	-0.1237
3.77	-6.8	0.0851	-0.1535

Mugs, Base 2			
y (mm)	z (mm)	dy_couple (mm)	dz_couple (mm)
2.36	-10.8	0.0533	-0.2439
0.13	-12.28	0.0029	-0.2773
-0.53	-13.96	-0.0120	-0.3152

Mugs Overall				
	Average dy_couple (mm)	Standard Deviation (mm)	Average dz_couple (mm)	Standard Deviation (mm)
Base 1	0.1310	0.0337	-0.1208	0.0280
Base 2	0.0148	0.0279	-0.2788	0.0291
Both	0.0729	0.0658	-0.1998	0.0840

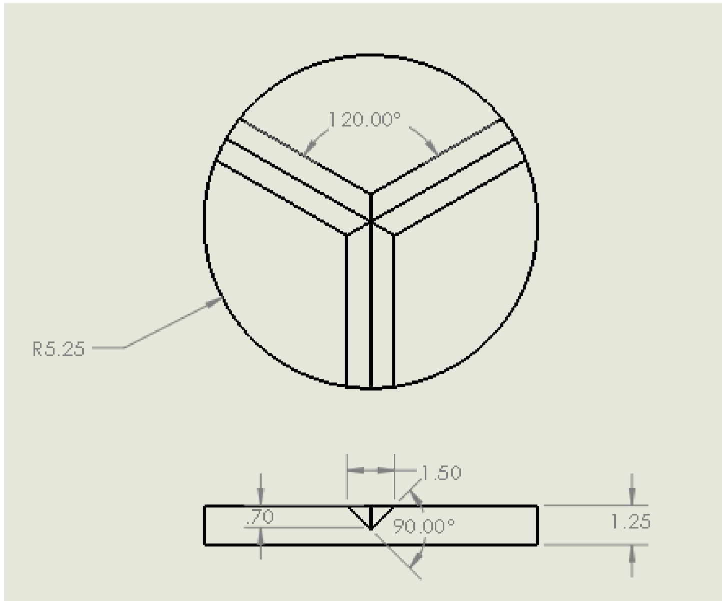
There is nearly a 10x – 100x difference between calculated and measured effort. This order of magnitude difference probably comes from the imprecision in the characterization procedure. Although care was taken to make sure that the tea set did not move too much during repeated loadings, the base may have shifted during reapplication of the load. Also, the laser exhibited quite a bit of diffraction and the calipers that were used to measure distances had accuracy of 0.01 mm, so the amount of human judgement used to determine the center of the beam and what to measure with the calipers probably induced quite a bit of error. In addition, the weights used for loading in the spreadsheet analysis were pure estimates and were probably not applied solely in the z direction, which may account for the y displacement that we measured as well. Since this is not a high accuracy application, the large discrepancies are ok since we still have < 1 mm deflection.

5. Conclusions and Future Directions

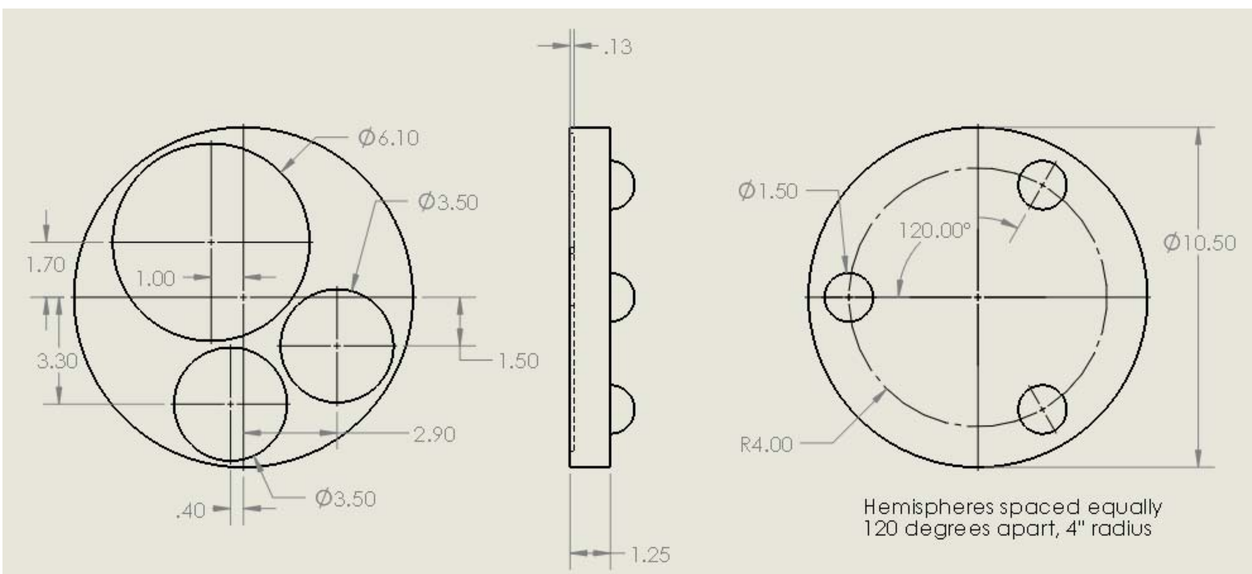
Overall, I'm pretty pleased with my work on this kinematic coupling. I learned a lot about how kinematic couplings work, learned how valuable jigs are for speeding up fabrication and how to use Abbe errors to help characterize a system. I made a teaset that I'm very proud of and am excited to give this to my friend.

For the future, I should definitely have a better understanding of what tools and materials are available in the design progress. Although I did do my due diligence by going online and attending office hours, I could have been more proactive in asking what exactly was available so I did not have to have as many design steps. In addition, the laser method only characterizes y and z displacement but not rotational displacements or x. Further characterization would be needed to fully verify that the kinematic

Appendix A: Technical Drawings
I: Initial Design

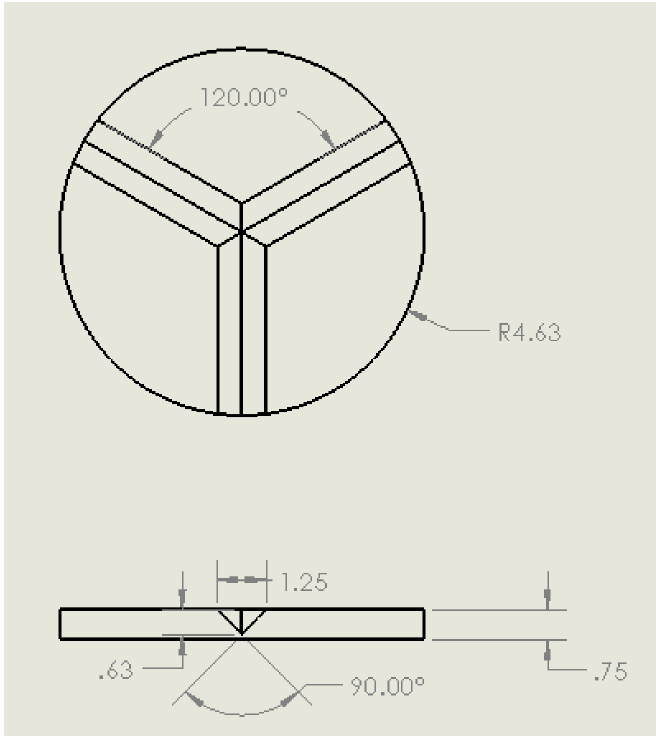


Initial bottom plate

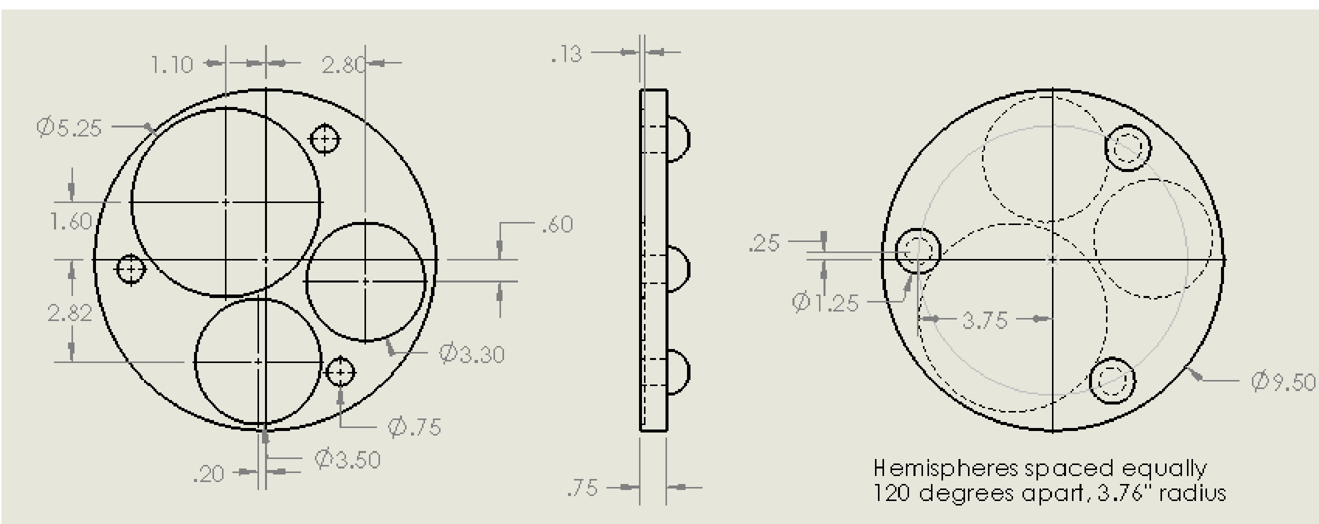


Initial top plate

II: Post-Feedback Design



Post-feedback bottom plate



Post-feedback top plate